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On Straddling Birth Intervals

AN interval, which contains a certain point of time *T* (e.g., survey point) is said to 'straddle' that point of time *T*. Luis Henry (1961) introduced the concept of straddling birth intervals (intervale de cheval) to estimate fertility of "subsequently fecund" women of a particular age group, where the age group is so chosen that a woman had at least one birth before the start of the age group, and the width of the age group was greater than the largest birth interval, i.e., — according to Henry—at least five years. He defined "subsequently fecund" women as those who would give birth to at least one more child before the end of her reproductive age.

Sehgal (1971, 1972) used a computer simulation approach to determine the sensitivity and robustness of birth intervals under different ascertainment plans, and found that straddling intervals would be useful as sensitive index of fertility change. By its very definition, however, straddling intervals cannot be determined at survey point, since the interval is not closed at survey point. To use it as an index of fertility change, therefore, requires it to be estimated, using other kinds of birth intervals (e.g., open, prospective closed, retrospective closed, interior, etc.), or their determinants, or any combination of these.

Straddling interval, of necessity, is a closed interval, but has a different distribution than that for all closed interval, because for all women, who have had i'-th birth at time *A*, and subsequently have (i + l)-th birth, the i-th closed birth interval can end anywhere in *(A, A* + 360) months, (actually, from *A* + post-partum amenorrhea+gestation, to end of reproductive age), but a straddling interval can end only in *(T,* end of reproductive period). Thus, their distribution is same as that of closed birth interval truncated on the left at time *T.*

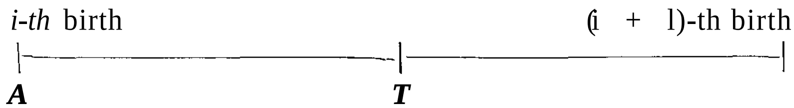


Chart 1. Straddling Birth Interval

Sheps and Menken (1970) derived theoretical distributions of various ascertainment plans, including straddling and closed intervals. These distributions, however, were derived for cohorts and stable populations, using general probability distributions for various competing risks, and more work needs to be done using specific probability density functions based on empirical distribution for competing risks like marriage, death, widowhood, and divorce, etc., before these can be used for estimating moments of the birth intervals.

Notations

Let

a = age at last live birth

$p_i(a, a+y)$ = fecundability of a woman aged $a+y$, with parity i, and open birth interval y.

This group of women is homogeneous in fecundability.

Let

X_i = random variable denoting the length of post-partum amenorrhoea period, following $(i+1)$-th live birth.

For sake of brevity, the phrase, "random variable denoting the length of", will be abbreviated as "*r.v*" (random variable).

$G = r.v.$ gestation period

Assumption 1 : For a live birth conception, $G = 9$ with probability 1.

Assumption 2 : No fetal losses, resulting in no abortions (induced or spontaneous) or still births.

$Z_i = r.v.$ waiting time for a fecund woman to i-th conception.

$B_i = r.v.$ i-th closed Birth Interval.

$S_i = r.v.$ i-th Straddling Interval.

$U_i = r.v.$ i-th Open Birth Interval.

$q_i(a, b) = 1 - p_i(a, b)$

Now, considering time as discrete,

Birth Interval = 9 months gestation + $(X_i=x)$ + $(Z_i=z)$

Therefore, $E(B_i) = 9 + E(X_i) + E(Z_i)$

where Z has a geometric distribution, i.e.,

$$E(Z_i) = \sum_{z=0}^{\infty} z\,[p_i(a+z, z) \prod_{j=0}^{z-1} q_i(a+j, j)]. \quad (1)$$

since $\qquad p_i(a+y, y) = 0 \quad \text{for} \quad y \leqslant x.$

Assumption 3: In the simple case,

$$p_i(a, b) = p_i(a) \text{ for all } a \text{ and } b$$

Then,
$$E(Z_i) = \sum_{z=0}^{\infty} z\, p_i(a) \quad q_i^{z-1-x}(a)$$

$$= p_i(a) \cdot \frac{1}{p_i^2(a)} = 1/p_i(a). \tag{2}$$

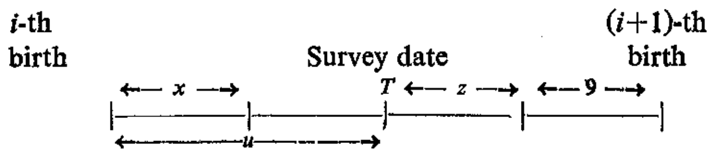
$$\therefore \qquad E(B_i) = 9 + E(X_i) + 1/p_i(a) \tag{3}$$

Straddling Interval

As stated earlier, a straddling interval, S_i is same as a closed interval, except that the interval ends only after survey time T, even though the woman may be pregnant at time T. Thus B_i has to be modified.

(*i*) *For woman not pregnant at survey time*:

(*a*)



Obviously, for $u \geqslant x$

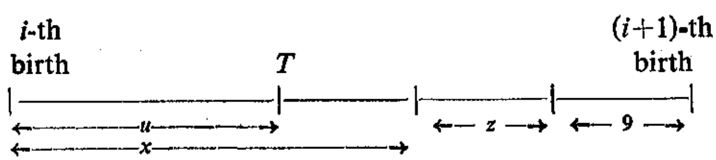
$$E(S_i) = E(U_i) + E(Z) + 9$$
$$\geqslant E(B_i). \tag{4}$$

Proportion of women in this group among all women of parity

$$i = \pi_{1i} = q_i^9(a). \tag{5}$$

This is the probability of no conception in last 9 months, knowing that earlier conception between i-th birth and $(T-9)$ months is not permissible, because, then she would be of parity $(i+1)$ and move to the next group:

(*b*)



In this case, $u < x$

and $$S_i = s = x + z + 9 = B_i. \tag{3a}$$

Assumption 5: U_i is distributed as normal with mean

$E(U_i)$ and variance σU_i^2

Proportion of women who are not pregnant at T, and for whom $u < x$ is given by

$$\pi_{2i} = P_r[U_i < X]. \tag{6}$$

Now

$$Pr[U_i<X] = E[Pr(U_i<x \mid X=x)],$$

$$= E\left[\Phi\left(\frac{x - E(U_i)}{\sigma U_i} \,\middle|\, X = x\right)\right], \tag{7}$$

where

$$\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy. \tag{8}$$

Now, $x > 0$, and with a probability of more than .99, the confidence intervals $(0, E(X_i)+3\sigma X_i)$ would contain the value of $X = x$. In this short range, the normal distribution function is nearly linear, and can be approximated by

$$\Phi(m) = c_1 + c_2 m$$

where c_1 and c_2 are constants.

Then, $$E(\Phi(m)) = c_1 + c_2 E(m) = \Phi(E(m))$$

Thus, equation (7) becomes,

$$\pi_{2i} = Pr[U<X] = \Phi\left[E\left\{\frac{x-E(U_i)}{\sigma U_i}\right\}\right]$$

$$= \Phi\left[\frac{E(X)-E(U_i)}{\sigma U_i}\right]. \tag{9}$$

Any errors due to this approximation are likely to be negligible.

(ii) *For women pregnant at survey time T* :

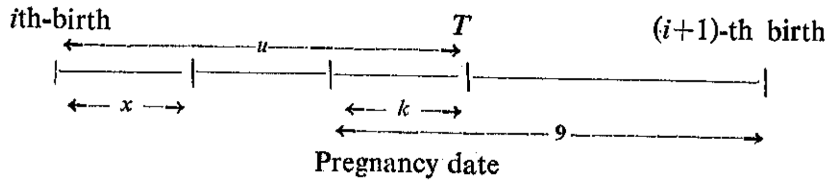
Proportion of pregnant women

$$\pi_{3i} = 1 - \text{proportion of women not pregnant}$$

$$= 1 - q_i^9(a) - \Phi\left[\frac{E(X)-E(U_i)}{\sigma U_i}\right]$$

$$= 1 - \pi_{1i} - \pi_{2i}. \tag{10}$$

(*c*) $u \geqslant x + 9$



In this case,

$$S_i = u + 9 - k, \qquad 0 < k < 9$$

and
$$E(S_i) = E(U_i) + 9 - E(K)$$

It is reasonable to assume a rectangular distribution for K and therefore,

$$E(K) = \int_0^9 \frac{k}{9}\, dk = 4.5$$

and
$$E(S_i) = E(U_i) + 4.5, \tag{11}$$

$E(K)$ can also be estimated, alternatively, from empirical data. Proportion of cases among all pregnant women,

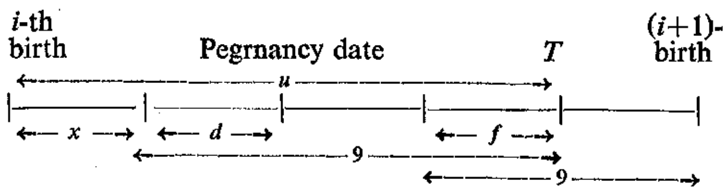
$$= Pr\,[U_i \geqslant X + 9]$$

$$= E[Pr\,(U_i \geqslant x + 9/X = x)]$$

$$= 1 - \Phi\left[\frac{E(X)+9-E(U_i)}{\sigma U_i}\right]. \tag{12}$$

(*d*) In this case,

$$u < x + 9$$



Here, $U = x + 9 - d$, A pregnant women at T could have become pregnant anywhere from $(x+d)$ months after i the birth to time T, an interval of less than 9 months (equals 9-d months). A correction is, therefore, called for in the expected value of the straddling interval, computed by equation (11).

Again, assuming a uniform distribution for D, we have,

$$E(D/X = x) = \int_0^x \frac{1}{x}\, v \cdot dv = \frac{x}{2}$$

$$\therefore \qquad E(D) = [E(D \mid X=x)] = \frac{E(X)}{2}. \tag{13}$$

$$E(F \mid D=d) = \text{Length of the interval from pregnancy date to survey date } T$$

$$= \int_0^{9-d} \frac{1f}{9-d}\, df = \frac{9-d}{2}$$

and,
$$E(F) = \frac{9 - \frac{E(X)}{2}}{2} = 4.5 - \frac{E(X)}{4}$$

giving

$$E(S_i) = E(U_i) + 9 - E(F)$$

$$= E(U_i) + 4.5 + \frac{1}{4} E(X). \tag{14}$$

Proportion of all pregnant women in this group is given by

$$\pi_{5i} = 1 - \pi_{4i}. \tag{15}$$

Estimation of Mean Length of Straddling Birth Interval

Combining equations (4), (3a), (11) and (14) with their corresponding probabilities, we get

$$E(S_i) = \pi_{1i}\,(E(U_i) + E(Z_i) + 9) + \pi_{2i}\,E(B_i)$$
$$+ \pi_{3i}\left[\pi_{4i}(E(U_i)+4.5) + \pi_{5i}\left\{ E(U_i) + 4.5 + \frac{1}{4} E(X)\right\}\right]$$
$$= \pi_{1i}[E(U_i) + E(Z_i) + 9] + \pi_{2i}\,E(B_i)$$
$$+ \pi_{3i}\left[E(U_i) + 4.5 + \frac{1}{4}\,\pi_{5i}\,E(X)\right]. \tag{16}$$

Estimation of π_{1i}

We know

$$E(B_i) = E(X_i) + E(Z_i) + 9$$

If $E(X_i)$ is used based on empirical data, we get

$$E(Z_i) = E(B_i) - E(X_i) - 9$$

Since
$$E(Z_i) = \frac{1}{P_i^{(a)}}$$

we get,
$$P_i^{(a)} = \frac{1}{E(Z_i)}$$

and
$$\pi_{1i} = q_i^9\,(a) = \left[1 - \frac{1}{E(Z_i)}\right]^9. \tag{17}$$

Variance of the Estimate of Straddling Birth Interval

As for equation (16), we have

$$E(S_i^2) = \pi_{1i}\, E\,[U_i + Z_i + 9]^2 + \pi_{2i}\, E(B_i^2)$$

$$+ \pi_{3i}\left[\pi_{4i}\, E(U_i+4.5)^2 + \pi_{5i}\, E\left(U_i+4.5+\frac{1}{4}\, X\right)^2\right].$$

$$= \pi_{1i}\,[E(U_i^2) + E(Z_i)^2 + 2E(U_iZ_i) + 18E(U_i) + 18E(Z_i) + 81]$$

$$+ \pi_{2i}\, E(B_i^2) + \pi_{3i}\left[E(U_i+4.5)^2\right.$$

$$\left. + \frac{1}{2}\,\pi_{5i}\, E(U_iX+4.5X) + \frac{\pi_{5i}}{16}E(X^2)\right] \qquad (18)$$

σ_{si}^2 can be, therefore, obtained from Eqs. (16) and (18).

Application

A computer simulation approach was used to determine the different ascertainment plans of birth intervals. The data by subjecting cohorts of 500 women (all married at age 15) to various competing risks, and open closed and straddling intervals were obtained, along with their variances, by age and parity of the mother. The results for two separate cohorts are given in Table 1.

In this table, $E(U_t)$, $E(T_t)$, $E(Zi)$, etc., have been abbreviated as U_i, T_i and Z_i, etc. Column 16 gives the estimated straddling birth intervals, while column 17 contains the corresponding observed straddling birth intervals in the simulated populations. A comparison shows that the estimates compare favourably with the observed values.

Conclusions

In this study, it has been assumed that there occur no fetal losses, resulting in no abortions or still births. Relaxation of this and some other assumptions would result in more complicated expressions for the estimate of mean and variance of the straddling birth interval.

Straddling birtn interval, not being observable at any survey point, has been estimated in this study as a function of open and closed birth intervals, and therefore, can be used as a sensitive measure for any changes in fertility, both in a contracepting as well as non-contracepting populations. Since the open and closed birth intervals can be ascertained at the survey time, the expressions in this study for estimates of mean and variance of straddling birth intervals can be utilized.

TABLE 1
ESTIMATION OF STRADDLING BIRTH INTERVALS

Parity	U_i	σU_i	T_i	Z_i	$q_i(a)$	$q_i^9(a)$	U_i+Z_i+9	π_{2i}	π_{3i}	π_{5i}	$U_i+Z_i+\frac{X_i}{4}\pi_{5i}$	=(7) ×(8)	=(4) ×(9)	=(10) ×(12)	Estimated S_i= (13) (14) (15)	Observed S_i
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Population I																
Age 25 years																
2	28.1	26.1	24.4	10.4	.9038	.4365	47.5	.1881	.3454	.2941	32.97	20.73	4.59	12.38	37.70	34.91
3	24.8	14.9	27.2	13.2	.9242	.4920	47 0	.0919	.4161	.2342	29.59	23.12	2.50	12 31	37.93	37.11
4	18.7	11.0	28.5	14.5	.9310	.5255	42 2	.1066	.3679	.3347	23.62	22.18	3 04	8 69	33.91	33.92
5	10.8	8.7	28.9	14.9	.9329	.5352	34.7	.2524	.2124	.6436	16.10	18.57	7.29	3.42	29.28	29.28
Population II																
Age 25 years																
2	28.3	30.5	24.4	10.4	.9038	.4365	47.7	.2224	.3411	.3196	33.20	20.82	5.43	11.32	37.57	37.08
3	24 9	26.4	27.2	13.2	.9242	.4920	47.1	.2254	.2826	.3398	29.82	23.17	6.13	8.43	37.73	38.38
4	18.3	10.7	28.5	14.5	.9310	.5255	42.8	.1069	.3676	.3413	23.23	22.49	3.05	8.54	34 08	33.03
5	11.1	8.2	28.9	14.9	.9329	.5352	35.0	.2284	.2364	.6383	16 40	18.73	6.60	3.88	29.21	30.97

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